

Non-compensability in composite indicators: a robust directional frontier method

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Abstract This paper follows the research mainstream aimed to link the efficiency frontier approaches [13] and the composite indicators (CI) methods [28]. More in detail, the main drawbacks of the CI methods based on Benefit of the Doubt (BoD) approach are the sensitivity to the outliers, the non-compensability and the lack of consideration about the marginal rate of substitution between simple indicators. Following [32] results, we propose a weighting method that bypassing all previous shortcomings suggests a comprehensive approach to construct robust and non compensatory composite indicators. This approach is based on the integration of BoD by a directional distance function.

Key words: Composite indicators, Non-compensability, Directional distance functions, Benefit of the Doubt.

1 Introduction

Composite indicators (CI) methods are increasingly recognized as a useful tool in policy analysis and public communication [28] for a variety of policy matters such as public units benchmark, industrial competitiveness, sustainable development, quality of life assessment, intensity of innovation.

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Nevertheless the Joint Research Centre of European Commission asserts that “*no uniformly agreed methodology exists to weight individual indicators before aggregating them into a composite indicator*”¹.

Several weighting techniques, in fact, has been proposed in literature, derived both from statistical methodologies, such as factor analysis, Data Envelopment Analysis (DEA) and Unobserved Components Models (UCM), and from more specific methods like multi-criteria evaluation (see e.g. [26]), Budget Allocation Processes (BAP), Analytic Hierarchy Processes (AHP), Conjoint Analysis (CA) or Method of Penalty in the Coefficient of Variation (MPCV) [14].

These weighting techniques have been applied in many socio-economic areas, like the evaluation of public policies (see e.g. [29] and [37] for local government performance evaluation; [33], for the European labor market analysis; [8] for social inclusion policies at EU level and [34] for government agencies evaluation). They have been used also in analysis of country performance through macroeconomic indicators (see e.g. [23], [24], [6] and [7] for internal market policies), for the construction of a environmental and ecological performance indicator (see e.g. [39], [1], [22], [31], [30] and [38]) and finally for the calculation of the Human Development Index (see e.g. [25], [15], [16] and [9]).

Several research papers concerning the comparison among different weighting methods provide interesting results, but these findings cannot be generalized especially because of the specific domains of the applications and because these approaches often produce *closed systems* in which the conclusion is the argument that has to be demonstrate.

In general, in the CI framework, there are no functional relationships among single indicators covering different aspects of a specific economic or social phenomenon and it cannot be assumed nomic causality [2], meaning that cannot be assumed certain or probabilistic general function covering relationship among instances. Moreover, in a nonparametric perspective, it is not even useful to introduce constraints, parametric functional forms or penalties which may be linked to a specific theoretical model because results would be clearly linked to the theoretical model that has generated them.

From a logical perspective, therefore, the claim of “*back to the details*” suggested by [28] appears as misleading, because the resulting composite indicator is clearly linked to the model that generated it.

This happens because, especially in this context, “*in the mathematical knowledge the consideration [the assumptions] is an operation that, for the objects, comes from outside; so it follows that the real object is altered*”² [20]. Knowledge, considered as a bias with respect to the expected measure, and the measure itself influence the analysis, for which there are no neutral actions without consequences: or the thought explains the evolution of the object (*adaequatio intellectus ad rem*), or the object itself is deformed by the thought (*adaequatio rei to intellectus*).

¹ http://composite-indicators.jrc.ec.europa.eu/S6_weighting.htm

² In Italian “*Nel conoscere matematico la considerazione è un operare che, per la cosa, vien da fuori; ne segue quindi che la cosa vera viene alterata*”.

Given that premise, the choice among different weighting functions, models or evaluation frameworks can be done or in an axiomatic way or according to the required statistical properties: in our paper, we suggest to highlight the statistical properties in general, and our solution in particular, rather than compare it with the most common aggregation methods.

Aware that "*all models are wrong, some are useful*" [3], therefore, we propose a data driven framework rather than a parametric method or a model linked to the definitions of specific causalities among different dimensions. In particular we propose a flexible framework where the non-compensability is not imposed in a fixed direction, but is linked to the data themselves and can be imposed in many different directions.

The paper is structured as follows: the methodological proposal and the properties of the CI aggregation method are described in section 2, results based on simulated data are explained in section 3 and concluding remarks are summarized in section 4.

2 A methodological proposal

In the present section we first show the foundations of the BoD and directional BoD models illustrated in [35] and [18] and, afterwards, we present a comprehensive model that adds the properties of the robust frontier models to the directional ones; finally, our proposed model is further specified in the particular case when outliers affect the direction.

2.1 The nonparametric framework

Our proposal is based on the logical union of two different proposals that aim, in the context of BoD nonparametric frontier models, to bypass two crucial drawbacks: on the one hand the lack of estimates robustness - see Robust BoD proposed by [35] - and, on the other hand, the full compensability among simple indicators - see directional BoD models proposed by [18].

As highlighted by [36], the classical BoD model can be regarded as an output-oriented CRS-DEA³ model, with all questionnaire items considered as outputs ($Y_q \in \mathbb{R}_+, \forall q = 1 \dots Q$) and a single input equal to one for all observations.

Therefore the production set of a CRS-DEA model, in the BoD framework as a particular case, can be written as:

$$H(\mathbf{1}, \mathbf{y}) = \text{Prob}(X \equiv \mathbf{1}, \mathbf{Y} \geq \mathbf{y}) \quad (1)$$

³ CRS-DEA: Costant Return to Scale Data Envelopment Analysis - see [5].

where Ψ is the support of $H(\mathbb{1}, \mathbf{y})$:

$$\Psi = \left\{ (\mathbb{1}, \mathbf{y}) \in \mathbb{R}_+^{1+Q} \mid H(\mathbb{1}, \mathbf{y}) > 0 \right\}. \quad (2)$$

Having defined the support Ψ and the random set $H(\mathbb{1}, \mathbf{y})$ it is possible to calculate the Farrel-Debreu (output) efficiency score as:

$$\lambda(\mathbb{1}, \mathbf{y}) = \sup \{ \lambda > 0 \mid H(\mathbb{1}, \lambda \mathbf{y}) > 0 \} \quad (3)$$

and, consequently, the BoD CI as the reciprocal of $\lambda(\mathbb{1}, \mathbf{y})$.

Composite indicators that are calculated through equation (3) satisfy the following desirable properties:

1. Weights are **endogenously** determined by the observed performances and the benchmark is not based upon theoretical bounds, but on a linear combination of observed best performances;
2. BoD CI is **weak monotone**: *Let CI an unbalanced-adjusted aggregation function of Q simple indicators I_q , CI is weakly positive monotone if for each $c > 0$, $CI(I_1, \dots, I_q, \dots, I_Q) \leq CI(I_1, \dots, I_q + c, \dots, I_Q)$;*
3. Weighting scheme is the **highest as possible**; this property is particular *"useful in policy arena, since policy-makers could not complain about unfair weighting: any other weighting scheme would have generated lower composite scores"* [28].

BoD model is usually based on the strong assumption of compensability among different simple indicators, but in practical applications very often exists a preference structure. In order to avoid this drawback, [18] suggests to include in the classical BoD model - equation (3) - a directional penalty using a *directional distance function*.

Using the BoD notation, the composite indicator can be calculated as the reciprocal of the directional distance function $D(\mathbb{1}, \mathbf{y}; g_y)$:

$$D(\mathbb{1}, \mathbf{y}; g_y) = \sup \{ \beta \mid (\mathbb{1}, \mathbf{y} + \beta g_y) \in \Psi \} \quad (4)$$

where g_y is the directional vector.

The directional BoD adds the following properties to the classical BoD model:

4. **non-compensability property**: the directional vector g_y rewards units along a generic direction (not only along the bisector direction) by penalizing units far from the chosen direction; therefore full compensability can be seen as a special case when $g_y = \mathbb{1}$.
5. **Translation property**: Directional BoD is invariant to the chosen mean normalisation method *i.e.* $D(\mathbb{1}, \mathbf{y} + \alpha g_y; g_y) = D(\mathbb{1}, \mathbf{y}; g_y) - \alpha$ for $\alpha \in \mathbb{R}_+$.

2.2 Robust directional BoD with a generic direction

Models highlighted in section 2.1 still suffer of a serious drawback: the lack of robustness with respect to outliers. To overcome this significant shortcoming the production set Ψ can be translated in a probabilistic framework following [12]'s proposal; considering a sample of m random variables with replacement $S_m = \{\mathbf{Y}_i\}_{i=1}^m$ drawn from the density of $\mathbf{Y}$, we define the random set $\tilde{\Psi}_m$ as:

$$\tilde{\Psi}_m = \bigcup_{j=1}^m \{(\mathbf{1}, \mathbf{y}) \in \mathbb{R}_+^{1+Q} | X \equiv \mathbf{1}, \mathbf{Y}_j \geq \mathbf{y}\}. \quad (5)$$

Therefore, the effect of an abnormal or outlier unit is damped given that the single unit is not compared to all the others, but to a sample subset of size m .

This generalization allows us to calculate iteratively the sample subset of size m (for $b = 1, \dots, B$ times) and for each b iteration the directional distance for the single unit from the maximum values can be defined as:

$$\tilde{D}_m^b(\mathbf{1}, \mathbf{y}; g_y) = \sup\{\beta | (\mathbf{1}, \mathbf{y} + \beta g_y) \in \tilde{\Psi}_m\}, \forall b = 1, \dots, B \quad (6)$$

More specifically equation (6) can be practically computed, in a multivariate setting, as suggested by [11], by using the dimensionless transformation by minimum for the $\mathbf{y}^* = g_y \cdot \mathbf{y}$:

$$\tilde{D}_m^b(\mathbf{1}, \mathbf{y}; g_y) = \max_{j=1, \dots, m} \left\{ \min_{q=1, \dots, Q} \left(\frac{Y_{jq}^{*b}}{y_{\cdot q}^*} \right) \right\} \quad (7)$$

and the order- m directional distance estimator $D_m(\mathbf{1}, \mathbf{y}; g_y)$ as:

$$D_m(\mathbf{1}, \mathbf{y}; g_y) = E(\tilde{D}_m^b(\mathbf{1}, \mathbf{y}; g_y)). \quad (8)$$

Following [4], finally, we can approximate the order- m directional distance estimator - even in a Shepard formulation - by computing the empirical mean over B :

$$\hat{D}_m(\mathbf{1}, \mathbf{y}; g_y) = 1/B \sum_{b=1}^B \tilde{D}_m^b(\mathbf{1}, \mathbf{y}; g_y) \quad (9)$$

The Robust directional BoD adds the following property to the classical BoD model and to directional BoD:

6. External robustness property: Robust directional BoD allows to remove outliers influence on the estimated frontier and, as a consequence, on the resulting CI ranking.

For seek of clarity, section 3 illustrates a graphical representation of the method using simulated data, in order to highlight that units' scores are not affected by outliers.

2.2.1 Robust PCA directional BoD

In the previous section a generic direction g_y has been included in the Robust directional method proposed.

For practical computation, however, the direction g_y , can be seen, for example, as the marginal rate of substitution among indicators; following this criteria, in case of additional information, it may be imposed by the researcher or derived directly from the data, for example through a Principal Components Analysis (PCA) as in [18].

PCA, in fact, lets to calculate the first principal component with the largest variance showing the internal structure and the main pattern of the data with the advantages of ease of calculation.

Despite this attractive feature, however, PCA models have several shortcomings; among others, all the classical PCA algorithms - based on least squares techniques - are set up on the assumption that outliers are not included in the dataset.

In order to bypass PCA outliers drawback, several robust versions of PCA have been developed by a modification of the covariance matrix (see *e.g.* [27]), by Projection Pursuit (see *e.g.* [21]) or by weighting Singular Value Decomposition (SVD) (see *e.g.* [19]).

In particular, in this application, we propose to use Robust PCA by Projection Pursuit because the more robust covariance matrix can be computationally intensive, especially if the involved covariance matrices must be estimated in a robust way (see in particular [10] and [17]).

This particular choice of g_y , based on PCA by Projection Pursuit, adds the following property to the Robust directional BoD:

7. Internal robustness property: Robust PCA directional BoD Robustness allows to remove outliers influence on the main direction estimated.

Simulation in section 3 highlights the robustness of the main direction estimated in presence of outliers.

3 Simulation

In the present section multiple simulations⁴ are introduced with the aim to test the validity of our approach and to illustrate the required properties of the suggested methods, rather than proof the stability towards another ones.

More specifically, we have generated a two-dimensional dataset of 300 units i with a marginal rate of substitution between first and second indicator equal to 0.5.

⁴ Simulations have been carried out thanks to *Compind* R package. This package can be requested from the authors.

$$\begin{cases} I_{i1} \in \mathcal{N}(0.5, 0.1) \\ I_{i2} = 0.5 \cdot I_{i1} + \mathcal{N}(0, 0.05) \end{cases}$$

In order to test the robustness of our measures, we have added to dataset an outlier set with a marginal rate of substitution between first and second indicator higher than 1.5.

More specifically, in the following figures, we have plotted the simulated simple indicators I_1 and I_2 with an increasing size and colour for higher values of CI; the composite indicator isoquants (light green for lower values of the composite indicator, dark green for higher values), are also displayed.

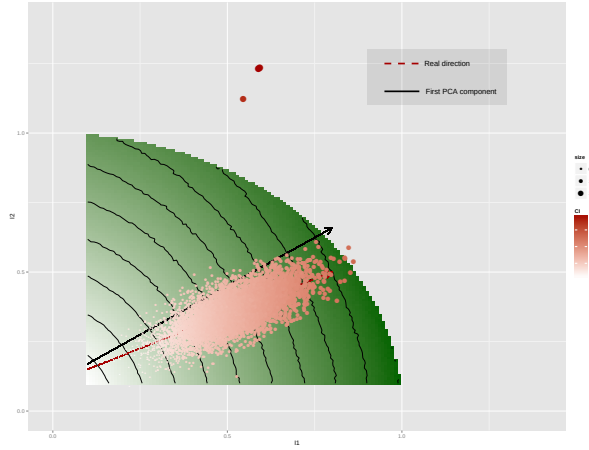


Fig. 1: Robust directional BoD on simulated data

Figure 1 confirm that selected approach is able to satisfy the requested properties outlined in section 2; in particular, it can be highlighted that - without imposing a priori compensability or non-compensability among indicators - directional measure lets to obtain a monotonic increasing of CI when I_1 or I_2 increase.

At the same time, it can be observed how outliers clearly influence (estimated rate of substitution equal to ...) the estimated direction g_y (internal robustness property) when classical PCA is used, while they don't have any impact on the frontier estimation (external robustness property) and on the relative ranking.

Finally, it can be noted that the CI is greater than 1 for the outlier units; this finding (in agreement with the order- m model) doesn't affect the scores of the other units and it is indicative of its characteristic.

With the aim to correctly estimate the real direction, we have applied the Robust PCA by Projection Pursuit to the same dataset obtaining an estimated rate of substitution equal to (see Figure 2) very similar to the real one (0.5).

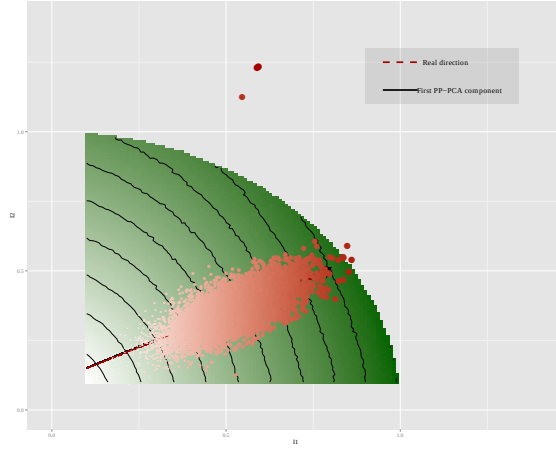


Fig. 2: Robust PCA (by Projection Pursuit) directional BoD on simulated data

4 Concluding remarks

This paper is included in the larger theme of the composite indicators construction. The authors have before dealt with this research sector and have produced several proposals of CI, in which the characteristics of robustness and non-compensability have been continually pursued. In this context a method based on Benefit of the Doubt technique has been developed and applied overall on infrastructure simple indicators. In progress, some improvements have been experimented; in particular: the order- m approach has been proposed in order to face the problem of outliers, and the directional approach has been integrated in the original BoD in order to enhance the characteristic of non-compensability of simple indicators.

The specific aim of this paper is to proceed on this research direction by the formulation of a comprehensive method of CI construction that is able to strengthen the mentioned characteristics, limiting as much as possible the usual drawbacks of CI: the sensitivity to the outliers, the non-compensability and the problem of the marginal rate of substitution between simple indicators. For this purpose, the paper has proposed a comprehensive model that adds the properties of the robust frontier with the directional ones. The paper is essentially methodological, even if a simulation is presented in order to illustrate the properties of the proposed method.

In summary, the method maintains the same peculiarity than in previous works, namely to be a data driven approach and to be based on data variability as principal criterion of construction. More specifically, the proposed method is characterized by the following properties: 1. Endogenous determination of the weights; 2. Aggregation function is weakly positive monotone; 3. Weighting scheme is the highest as possible; 4. Non-compensability of simple indicators; 5. Invariance of obtained CI to the normalization method; 6. Robustness because of outliers influence removal;

7. Further improvement of the robustness by means of directional BoD based on Principal Components Analysis.

The illustrated simulation introduces, beside the original two-indicators dataset, an outliers set characterized by the abnormal rate of substitution between those two indicators. The construction of the composite indicator by the way of the Robust PCA Directional BoD confirms that the proposed method assures as much as possible the characteristics of robustness and non-compensability.

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